Computer-Aided Design of Parametric Amplifiers

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Abstract—A computer-aided design approach is developed for the analysis and design of single-tuned parametric amplifiers. The voltage gain-bandwidth product ($\sqrt{G}B$) is used as a maximizing objective function for the optimum design. An 18-GHz single-tuned amplifier using a single-packaged varactor diode is designed as an illustration. The experimental amplifier exhibited a large $\sqrt{G}B$ of 2400 MHz and a low noise temperature of 180°K at room temperature ambient.

Introduction

MUCH WORK has been published on the bandwidth performance of multiple-tuned parametric amplifiers. In order to realize a broad passband with a multiple-tuned parametric amplifier, the corresponding fundamental single-tuned amplifier should be designed to obtain the maximum voltage gain–bandwidth product ($\sqrt{G}B$) obtainable with the given varactor diode. DeJager has calculated the maximum $\sqrt{G}B$ of the single-tuned parametric amplifier and the optimum idler frequency based on an equivalent circuit in which the package capacitance of a varactor diode is omitted and the idler rejection filter is ideal [1]. However, the practical design process of a single-tuned parametric amplifier usually involves considerable cut-and-try and is excessively time-consuming. In an effort to reduce these problems, Getsinger has suggested the utilization of a computer for the design of diode-using microwave components and has shown its effectiveness in the analysis and design of an X-band parametric amplifier [2].

In this paper, a computer-aided design approach is developed for the analysis and design of single-tuned parametric amplifiers. The $\sqrt{G}B$ is used as a maximizing objective function for the optimum design of the single-tuned amplifier which consists of a cascade connection of distributed transmission lines and a varactor diode. The design procedure has two parts: 1) formulation of the $\sqrt{G}B$ in terms of circuit parameters; and 2) optimization using a modified pattern search maximization procedure that does not require any derivatives [3].

One of the authors has reported an 18-GHz single-tuned parametric amplifier with a $\sqrt{G}B$ of 1450 MHz and a noise temperature of 245°K including a circulator loss at room temperature ambient [4]. The amplifier, using a pill-encapsulated varactor diode and a radial idler cavity, has been optimized experimentally. As an illustration, a single-tuned amplifier is designed and evaluated experimentally at 18 GHz. A newly developed packaged varactor diode, whose package capacitance is less than half of that for the pill-encapsulated diode, is used for the experimental single-tuned amplifier. The amplifier alone exhibited a large $\sqrt{G}B$ of 2400 MHz and a low noise temperature of 180°K at room temperature ambient. The required pump power was 30 mW or less at 73 GHz, making it easily possible to introduce a solid-state pump source. The calculated results of the double-tuned amplifier are also shown to estimate the potential bandwidth capability of the experimental amplifier.

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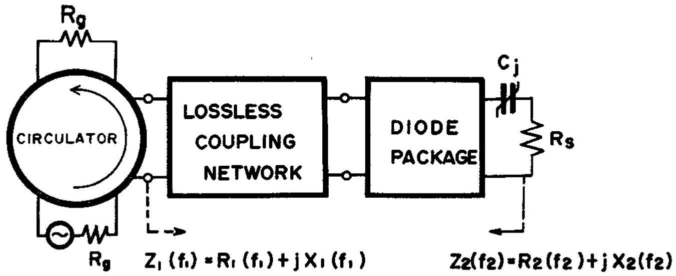

Fig. 1. Schematic of parametric amplifier using packaged varactor diode.

Computer-Oriented Design Formulation of Single-Tuned Parametric Amplifier

Fig. 1 is a schematic of the parametric amplifier which consists of a varactor diode, a lossless coupling network, and a circulator. A varactor diode itself is represented by a series connection of a variable junction capacitance C_j and a loss resistance R_s. However, varactor diodes have conventionally been mounted in small cylindrical packages whose parasitic effects sometimes deteriorate amplifier performances [5].

If the junction capacitance of a varactor diode is pumped at a frequency f_p and the diode, including an external circuit, is resonated at frequencies f_{10} and f_{20} $(=f_p-f_{10})$, the varactor diode exhibits a parametric negative resistance and an incident signal at f_{10} can be amplified. The f_{10} and f_{20} are midband values of the signal and the idler frequencies, respectively. In Fig. 1, $Z_1(f_1)$ and $Z_2(f_2)$ are the unpumped impedances of the signal and the idler circuits, respectively. If the total circuit is to operate as a parametric amplifier, the lossless coupling network should satisfy the following conditions.

1) The idler circuit must be resonated at the midband idler frequency f_{20}, i.e., $X_2(f_{20})=0$.

2) The signal circuit must be resonated at the midband signal frequency f_{10}, i.e., $X_1(f_{10})=0$.

3) If the idler energy propagates to the circulator,

the noise temperature of the amplifier increases. Therefore, the idler energy must be prevented from propagating out of the coupling network, i.e., $R_2(f_{20})=0$.

4) The coupling ratio of $R_1(f_{10})$ to the load resistance R_g must be held at the value determined by a desired midband gain. For instance, $R_1(f_{10})/R_g=1/(\tilde{Q}_1\tilde{Q}_2-1)$ under the high gain condition [6].

If these four conditions are satisfied, the normalized $\sqrt{G}B$ of the single-tuned parametric amplifier at high gain is given by [7]

$$\frac{\sqrt{G}B}{f_{10}} = \frac{2}{\dfrac{Q_{10}}{\tilde{Q}_1\tilde{Q}_2} + \dfrac{f_{10}Q_{20}}{f_{20}}} \tag{1}$$

where $\tilde{Q}_1$ and $\tilde{Q}_2$ are the dynamic quality factors of the varactor diode at f_{10} and f_{20}, respectively [6]. Q_{10} and Q_{20} are the unpumped unloaded quality factors of the signal and the idler circuits:

$$Q_{10} = \frac{f_{10}}{2R_1(f_{10})} \left.\frac{dX_1(f_1)}{df_1}\right|_{f_1=f_{10}} \tag{2}$$

$$Q_{20} = \frac{f_{20}}{2R_s} \left.\frac{dX_2(f_2)}{df_2}\right|_{f_2=f_{20}}. \tag{3}$$

The design objective here is to maximize $\sqrt{G}B/f_{10}$ under the four conditions. Since the optimum idler frequency for the maximum $\sqrt{G}B/f_{10}$ is mainly determined by the self-resonant frequency of a varactor diode, the noise temperature is sometimes higher than the minimum noise temperature obtainable with the given varactor diode. For most practical parametric amplifiers the noise temperature is one of the most important design factors and should be designed to be below the maximum allowable $(T_{e,m})$. Therefore, a fifth design condition, $T_e \leqq T_{e,m}$, must be added to the others. If the coupling network is lossless, the noise temperature T_e is given by [6]

$$T_e = \frac{1+\tilde{Q}_2^2}{\tilde{Q}_1\tilde{Q}_2-1} T_a \tag{4}$$

where T_a is the ambient temperature of the varactor diode.

Lossless Coupling Network

Coupling networks at microwave frequencies for single-tuned parametric amplifiers have been composed of cascade connections of distributed transmission lines. Typical models of coupling networks, shown schematically in Fig. 2, consist of transmission line elements, an idler rejection filter, and an ideal transformer. As the two types of idler rejection filter result in different bandpass characteristics and a different optimum idler frequency for the maximum $\sqrt{G}B$, it is important to determine whether the parallel or the series type is to be employed [8].

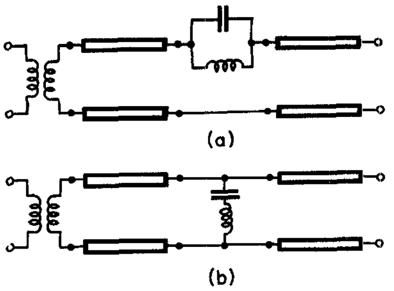

Fig. 2. Typical models of lossless coupling networks. (a) Parallel idler rejection filter. (b) Series idler rejection filter.

Coaxial transmission lines are useful for low-impedance signal circuits of parametric amplifiers because of their low insertion losses even at K-band frequencies. 18-GHz parametric amplifiers have been developed with coaxial transmission line signal circuits and have exhibited a low noise temperature [4], [7]. In the case of a coaxial line circuit, a parallel idler rejection filter is preferable because it is easily constructed with a short-circuited radial transmission line formed in the outer coaxial conductor. On the other hand, it is difficult to fabricate a series idler rejection filter with a coaxial transmission line, although it can be realized by a quarter-wave (at f_{20}) open-circuited stub.

Two transmission lines in conjunction with the idler rejection filter are used for resonating both the signal and the idler circuits. Their characteristic impedances and electrical lengths should be chosen at the optimum values so that the maximum $\sqrt{G}B$ may be obtained with the particular varactor diode being used. The ideal transformer can be approximately realized by a multisection stepped impedance transformer.

Varactor Diode

The choice of packaged varactor diode or waveguide wafer-type varactor diode (Sharpless diode) is one of the most important alternatives in the design of parametric amplifiers, especially at frequencies above 10 GHz [5]. The packaged diodes have several advantages, i.e., simple replacement and easy testing, but the parasitic elements associated with the package limit the realizable bandpass characteristics. At higher signal frequencies, the junction capacitance must be smaller and the parasitic elements must be as small as possible. Recently, varactor diodes with very small packages have become available (Sylvania, D5147J). A representative packaged varactor diode, shown in Fig. 3(a), exhibits a diameter of 1.145 mm and a height (dielectric portion) of 0.320 mm. A packaged varactor diode can be represented by a simplified equivalent circuit [9] as shown in Fig. 3(b). C_p is the package capacitance that can be inferred from the effects of the dielectric cylinder, and L_s is the lead inductance that arises from the inductances of the ribbon used to contact the diode chip. The manufacturer's quoted values of these parasitic elements are 0.11 pF and 0.15 nH. For the experimental single-

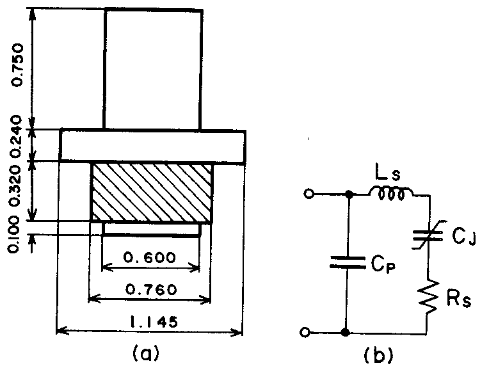

Fig. 3. Packaged varactor diode used and simplified equivalent circuit.

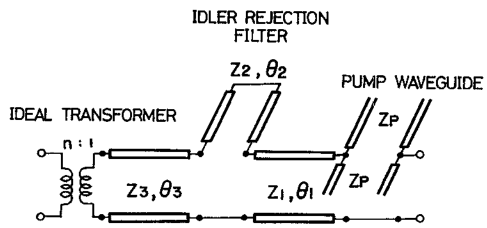

Fig. 4. Lossless coupling network for computation.

tuned amplifier, this packaged varactor diode was employed because of its potentially wide bandwidth. Other parameters of the varactor diode used are

$$f_{c-6} = 600 \text{ GHz} \qquad C_{j0} = 0.14 \text{ pF}.$$

Assumed that the junction capacitance C_j at a reverse bias voltage is 0.11 pF, the corresponding series and parallel resonant frequencies are estimated to be about 39 GHz and 55 GHz, respectively. The dynamic quality factor of the varactor diode can be obtained experimentally by cold tests [10], and the typical measured value of $\tilde{Q}_1$ at 18 GHz was, as shown subsequently, about 4.8 including circuit losses.

Numerical Results

Fig. 4 illustrates the coupling network used in the computations. The network consists of a pump transmission line, an idler rejection filter, two transmission lines, and a transformer. The transmission lines are assumed lossless and uniform, and the transformer ideal. In Fig. 4, the electrical lengths $\theta_i(i=1, 2, 3)$ are the values at the midband signal frequency of 18 GHz. The equivalent circuit parameters associated with junctions between adjacent lines are disregarded in this example.

A waveguide is useful for realizing the pump transmission line because it prevents propagating the power at frequencies below cutoff even without any rejection filters. If the signal and idler frequencies are below the waveguide cutoff, the characteristic impedance Z_p of the pump waveguide is inductive:

$$Z_p = j\frac{b}{a}\,\frac{240\pi}{\sqrt{(\lambda/\lambda_c)^2 - 1}} \tag{5}$$

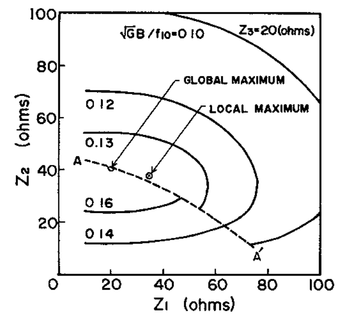

Fig. 5. Contour plots of normalized gain–bandwidth product for constant Z_3. Parameters used in the computations: $L_s=0.15$ nH, $C_p=0.11$ pF, $C_j=0.11$ pF, $\tilde{Q}_1=4.8$, and $f_{20}=55$ GHz.

where λ and λ_c are the free-space and the cutoff wavelengths, respectively, and a is the width and b the height of the waveguide.

Since line 2 is the idler rejection filter, the idler circuit is composed of line 1 and the varactor diode. Therefore, the electrical length θ_2 is determined independently of the characteristic impedance Z_2, from condition 3). The electrical length θ_1 can be determined as a function of the characteristic impedance Z_1 from condition 1). Once the characteristic impedances Z_1 and Z_2 are assumed, the electrical length θ_3 is obtained as a function of the characteristic impedance Z_3 from condition 2). Condition 4) is satisfied by adjusting the ideal transformer ratio n. Therefore, the independent variable parameters for this example are the characteristic impedances Z_1, Z_2, Z_3 and the midband idler frequency f_{20} and $\sqrt{G}B/f_{10}$ can be expressed using these parameters. The optimum design is performed so as to maximize $\sqrt{G}B/f_{10}$ by varying the parameters.

An 18-GHz single-tuned parametric amplifier was designed utilizing this procedure. In order to obtain the maximum $\sqrt{G}B/f_{10}$ and the corresponding parameter values, a pattern search maximization procedure was applied to function (1). Since the characteristic impedances among the parameters should be physically realizable, the following constraints were imposed on them:

$$Z_i \geqq 15, \qquad i = 1, 2, 3.$$

These constraints were taken into account during the maximization procedure by imposing a large penalty on the objective function when any constraint violation occurs. Since some discontinuities of $\sqrt{G}B/f_{10}$ developed on the response hypersurface, and the feasible region contained a local maximum as shown subsequently, the search sometimes terminated at a false maximum. In order to prevent obtaining a false maximum, several sets of initial values of the parameters were employed and the search was performed.

Fig. 5 shows a typical two-dimensional contour plot encountered in this problem. The contour plot represents the case in which the midband idler frequency f_{20} is 55 GHz and Z_3 is 20 Ω. The global maximum is obtained when the characteristic impedances Z_1 and Z_2 are about 20 Ω and 40 Ω, respectively, and is located at the

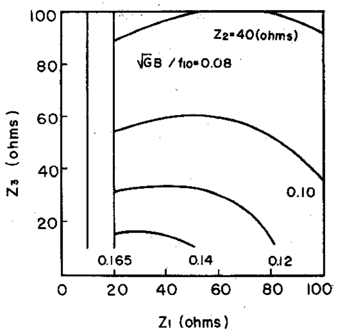

Fig. 6. Contour plots of normalized gain–bandwidth product for constant Z_2. Parameters used in the computations are the same as those used for Fig. 5.

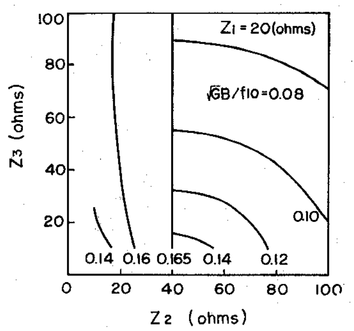

Fig. 7. Contour plots of normalized gain–bandwidth product for constant Z_1. Parameters used in the computations are the same as those used for Fig. 5.

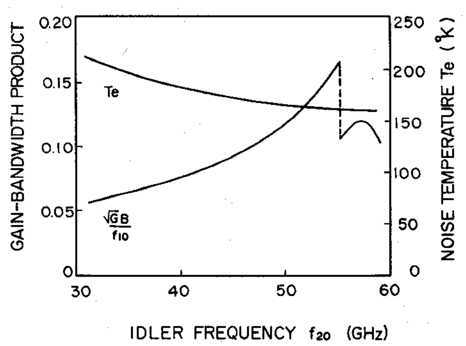

Fig. 8. Calculated noise temperature and maximum normalized gain–bandwidth product as a function of midband idler frequency. Varactor parameters used in the computations: $L_s = 0.15$ nH, $C_p = 0.11$ pF, $C_j = 0.11$ pF, and $\bar{Q}_1 = 4.8$.

discontinuity AA'. This corresponds to the fact that the maximum $\sqrt{GB}/f_{10}$ is obtained when the signal circuit is resonanted by the characteristic impedance Z_2 and the electrical length θ_3 is zero. Two-dimensional contour plots about the global maximum, shown in Figs. 6 and 7, indicate that it occurs independently of Z_3, or hence when θ_3 is zero.

Fig. 8 shows the calculated maximum $\sqrt{GB}/f_{10}$ and the noise temperature T_e versus the midband idler frequency f_{20}. The maximum $\sqrt{GB}/f_{10}$ changes suddenly when f_{20} is about 55 GHz. This discontinuity corresponds to that in the electrical length θ_1, determined from condition 1), obtained at the parallel resonance of the varactor diode at which point $\theta_1 = 0$ and the inductive impedance of the varactor diode changes into capacitive. It can be seen from Fig. 8 that the maximum $\sqrt{GB}/f_{10}$ of the single-tuned parametric amplifier is obtained when the midband idler frequency is chosen at about 55 GHz, the parallel resonant frequency of the varactor diode corresponding to $\theta_1 = 0$.

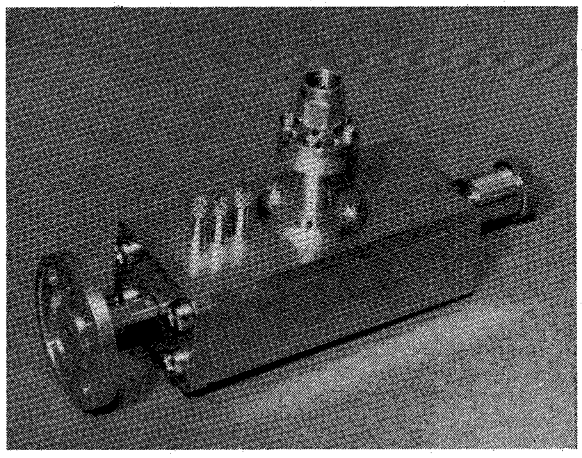

Fig. 9. Experimental single-tuned parametric amplifier.

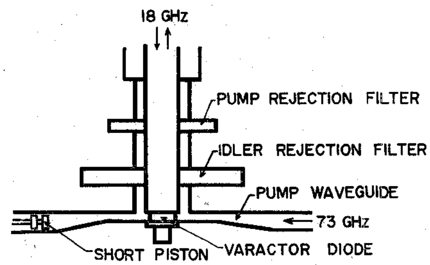

Fig. 10. Schematic diagram of experimental single-tuned parametric amplifier.

Experimental Results

A single-tuned parametric amplifier was fabricated to check the validity of the numerical results. The midband idler frequency was chosen at 55 GHz to maximize $\sqrt{GB}$. The experimental single-tuned amplifier is shown in Fig. 9. The signal power at 18 GHz is fed through the OSM connector. The pump power at 73 GHz is supplied to the varactor diode through a standard waveguide (WRJ-740) and matched with three stubs and a short piston as shown in Fig. 9.

A schematic diagram of the experimental single-tuned amplifier appears in Fig. 10. The signal circuit mainly consists of a coaxial line except for pump and idler rejection filters that are composed of short-circuited radial lines formed in the outer coaxial conductor. The varactor diode is mounted in a reduced height and width waveguide (0.3 mm by 2.5 mm) of cutoff frequency 60 GHz. The waveguide mount is connected to the standard pump waveguides through tapered sections. The signal-circuit tuning reactance is determined by the wave impedance, e.g., the height of the radial rejection filter. The section of the coaxial line between the idler rejection filter and the input connector forms a quarter-

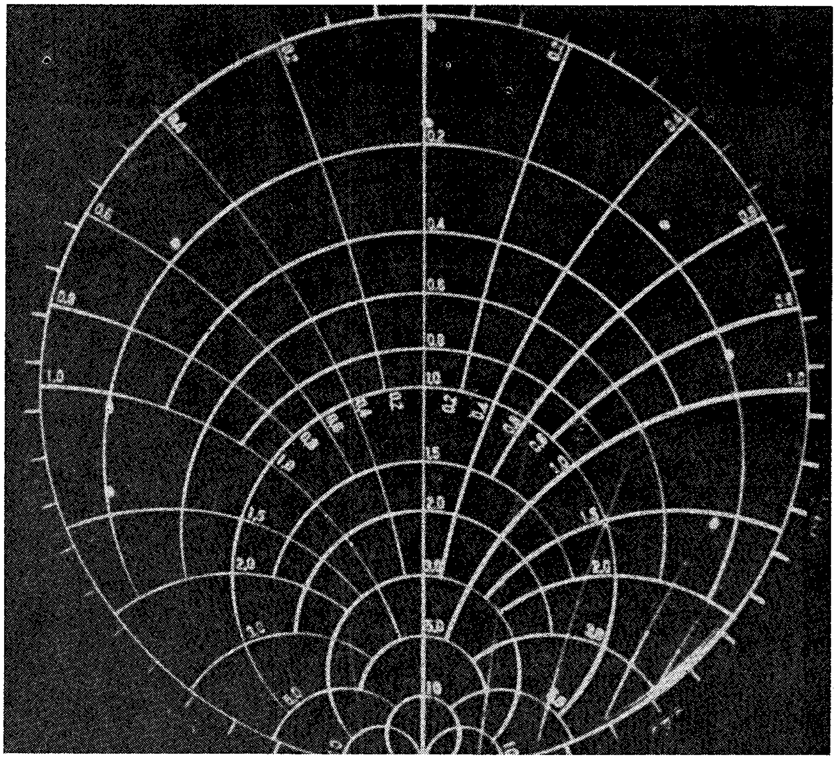

Fig. 11. Unpumped impedance locus of single-tuned parametric amplifier. Bias voltage are -9, -6, -3, -1, 0, 0.5 V and 10 μA in the clockwise direction.

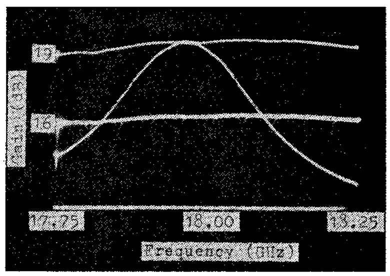

Fig. 12. Measured bandpass characteristic of experimental single-tuned amplifier. The varactor diode used in the amplifier is biased at -0.9 V. Markers indicate the frequency difference of 50 MHz.

TABLE I

SINGLE-TUNED AMPLIFIER PERFORMANCE

	Calculated	Measured
Signal frequency	18 GHz	18.0 GHz
Idler frequency	55 GHz	55.4 GHz
Dynamic Q at signal frequency		4.8
Coupling ratio	5.4	6.0
Unloaded Q of signal circuit	44	49
Unloaded Q of idler circuit	19	22[a]
Gain–bandwidth product	2700 MHz	2400 MHz
Noise temperature	165°K	180°K
Pump frequency	73 GHz	73.4 GHz
Pump power		30 mW

[a] Estimated from measured bandpass characteristic of experimental amplifier.

wavelength transformer including the pump rejection filter.

The dynamic quality factor $\tilde{Q}_1$ can be obtained from the unpumped impedance locus of the experimental single-tuned amplifier at the signal frequency. A typical measured impedance locus is shown in Fig. 11. The typical value of $\tilde{Q}_1$ is found to be about 4.8 including the losses of both the diode mount and the transmission line coupling network. It can also be seen from Fig. 11 that the experimental single-tuned amplifier satisfies the high gain condition. The measured bandpass characteristic of the amplifier, shown in Fig. 12, exhibits a 3-dB bandwidth of 270 MHz at a midband gain of 19 dB and hence a gain–bandwidth product $\sqrt{G}B$ of 2400 MHz. On the basis of "hot–cold" waveguide loads at room and liquid nitrogen temperatures, noise measurement indicated a noise temperature of 180°K for the amplifier alone at room temperature ambient. The calculated and measured performances of the experimental single-tuned amplifier are compared in Table I. The slight differences between the measured and the calculated values are due to the fact that the calculation did not take into account the parasitic circuit parameters of the transmission line junctions or the increase of the diode loss at the idler frequency. Moreover, the transformer was assumed ideal and its real frequency dependence property was neglected.

The incident pump power at 73 GHz required to drive the experimental amplifier was 30 mW or less after careful pump matching. Therefore, it is feasible to utilize a solid-state pump source and simplify the overall amplifier.

The unloaded Q of the idler circuit was estimated in Table I from the measured bandpass characteristics of

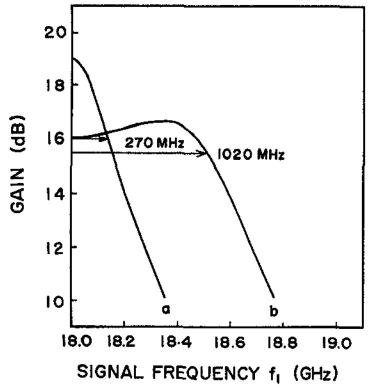

Fig. 13. Calculated bandpass characteristics. Curve a—single-tuned parametric amplifier: $Q_1Q_2=7.5$, $Q_{10}=49$, and $Q_{20}=22$. Curve b—double-tuned: $Q_p=10$.

the experimental single-tuned amplifier. A double-tuned amplifier is realizable by connecting a parallel resonant circuit across the signal circuit of the single-tuned amplifier [11], and its bandpass characteristics can be calculated by using the measured circuit parameters of the single-tuned amplifier. Fig. 13 shows the calculated double-tuned bandpass characteristic along with the optimum Q_p of the parallel resonant circuit. It is seen from Fig. 13 that a 1-dB bandwidth of 1000 MHz or more at 16-dB gain is theoretically achievable by introducing double tuning in the experimental single-tuned amplifier.

Conclusion

The computer-aided optimization of the design of a single-tuned parametric amplifier has been discussed. An 18-GHz single-tuned amplifier using a packaged varactor diode has been designed and characterized as an illustration. Although the design has not considered the transmission line junction effects, the calculated results have afforded a good prediction for the amplifier performance. The design procedure has been developed for an 18-GHz parametric amplifier, but the procedure can be applied to parametric amplifiers at any frequencies.

The problems remaining to be solved are the considerations of real properties of transmission lines, the measurement of the varactor diode equivalent circuit at millimeter-wave frequencies, and the development of an efficient optimization algorithm in the presence of some discontinuities and local optimums on the response hypersurface.

Acknowledgment

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